Homework 13 (due on 12/6)

- Read Section 31.
- Everything necessary to do this homework will be stated in class by the end of Monday December 2. If you're the sort of person who wants to do your homework earlier than that, note that the main statements you need to do problems in Section 26 and 31 are Theorems 26.4 and 26.5, Definition 31.2, and Corollary 31.4.

30.1 Find the following limit if it exists. (a) $\lim_{x\to 0} \frac{e^{2x}-\cos x}{x}$.

30.2 Find the following limit if it exists. (c) $\lim_{x\to 0}(\frac{1}{\sin x} - \frac{1}{x})$.

30.3 Find the following limit if it exists. (c) $\lim_{x\to 0^+} \frac{1+\cos x}{e^x-1}$.

30.5 Find the following limit if it exists. (a) $\lim_{x\to 0}(1+2x)^{1/x}$.

26.3 (a) Use Exercise 26.2 to derive an explicit formula for $\sum_{n=1}^{\infty} n^2x^n$.

26.4 (a) Observe that $e^{-x^2} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}x^{2n}$ for $x \in \mathbb{R}$, since we have $e^x = \sum_{n=0}^{\infty} \frac{1}{n!}x^n$ for $x \in \mathbb{R}$.

(b) Express $F(x) = \int_0^x e^{-t^2}dt$ as a power series.

26.6 Let $s(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$ and $c(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$ for $x \in \mathbb{R}$.

(a) Prove $s' = c$ and $c' = -s$.

(b) Prove $(s^2 + c^2)' = 0$.

(c) Prove $s^2 + c^2 = 1$.

Actually $s(x) = \sin x$ and $c(x) = \cos x$, but you do not need these facts.

31.1 Find the Taylor series for $\cos x$ and indicate why it converges to $\cos x$ for all $x \in \mathbb{R}$.