Exam 2

Name: ______________________________

Section: ______________ **Recitation Instructor**: ______________________

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 11.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand.
- You will be given exactly 90 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.

ACADEMIC HONESTY

- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page. You must indicate if you desire work on the back of a page to be graded.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions and statements regarding academic honesty: ______________________________

SIGNATURE

Standard Response Questions. Show all work to receive credit. Please $\boxed{\textbf{BOX}}$ your final answer.

1. (4 points) Find the most general antiderivative of $f(x) = 2\sec^2(x) - x^5$.

2. (4 points) Evaluate: $\displaystyle\int_1^4 \frac{t^3 + \sqrt{t}}{t^2}\, dt$.

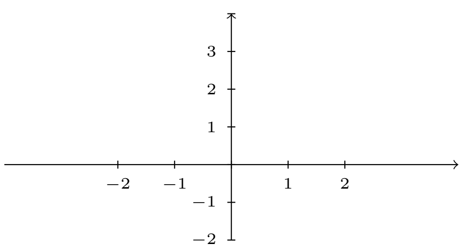
3. (6 points) Let $F(x) = \displaystyle\int_{\cos x}^{1} \sqrt{5 - t^2}\, dt$. Find $F'(x)$ ***without actually finding*** $\mathbf{F}(\mathbf{x})$.

4. (7 points) Find the absolute maximum and the absolute minimum values of

$$f(x) = \sin x + \cos x \qquad \text{on the interval } [0, \pi].$$

5. Let $f(x) = \begin{cases} x+2 & \text{if } -2 \leq x < 0 \\ x-1 & \text{if } 0 \leq x \leq 2 \end{cases}$

(a) (3 points) Sketch the graph of $f(x)$.



(b) (2 points) Use your sketch in part (a) to find the absolute maximum and absolute minimum values of f on $[-2, 2]$, if they exist.

Absolute Minimum=

Absolute Maximum=

(c) (2 points) If an absolute extremum does not exist, explain why does this not contradict the statement of the Extreme Value Theorem.

6. Suppose: $f(x) = \dfrac{x^2}{x-1}, \qquad f'(x) = \dfrac{x(x-2)}{(x-1)^2}, \qquad f''(x) = \dfrac{2}{(x-1)^3}$

(a) (1 point) What is the domain for f?

(b) (3 points) Write the equations for all vertical, horizontal, and slant asymptotes, if they exist.

(c) (4 points) Identify the intervals over which $f(x)$ is increasing / decreasing. *Write your answer in interval notation in the boxes below.*

f is increasing on:

f is decreasing on:

(d) (2 points) Identify all points (x, y) where f(x) attains its local maximum or minimum. *Write your answer in the boxes below.*

f has a local minimum at $(x, y) =$

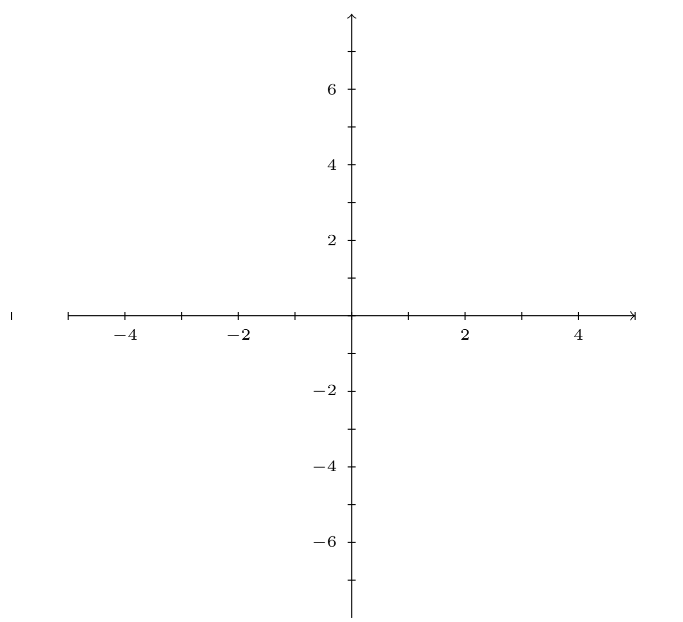
f has a local maximum at $(x, y) =$

(e) (2 points) Identify the intervals over which $f(x)$ is concave up / concave down. *Write your answer in interval notation in the boxes below.*

f is concave up on:

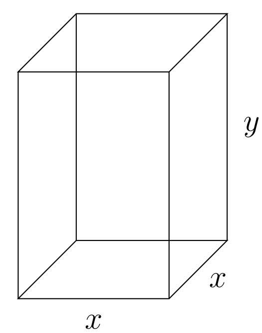
f is concave down on:

(f) (2 points) Sketch the curve of $y = f(x)$. *Parts (a)-(e) may be helpful.*



7. A total of 225 ft^2 of material is used to make a box with **no top** and a **square** base.

   (a) (6 points) Express the volume of the tank, V, only in terms of x, the length of the base.



$V(x) =$

   (b) (8 points) Find the maximum volume of such a tank for $x \in (0, 15]$.
   Include units! Use techniques of calculus to justify that your answer is a maximum.

Multiple Choice. Circle the best answer. No work needed. No partial credit available.

8. (4 points) Newton's method can be used to approximate $\sqrt[5]{33}$ by finding the root of which of the following functions?

   A. $f(x) = x^5 + 33$

   B. $f(x) = \sqrt[5]{x} - 33$

   C. $f(x) = \sqrt[5]{x} + 33$

   D. $f(x) = x^5 - 33$

   E. $f(x) = x - (33)^5$

9. (4 points) Using three equally-spaced rectangles of equal width, find the upper sum approximation of the area between the curve $y = \dfrac{1}{x}$ and the $x-$axis from $x = 2$ to $x = 8$.

   A. $\frac{13}{12}$

   B. $\frac{11}{4}$

   C. $\frac{11}{6}$

   D. $\frac{39}{24}$

   E. 3

10. (4 points) Using a linear approximation with $a = 64$, find an estimate of $\sqrt{63}$

    A. $8 + \frac{1}{16}$

    B. $8 - \frac{1}{8}$

    C. $8 - \frac{1}{16}$

    D. $8 + \frac{1}{8}$

    E. 8

11. (4 points) Suppose $\int_{-1}^{2} f(x)\ dx = 6$ and $\int_{-1}^{3} f(x)\ dx = 8.$ Find $\int_{3}^{2} 5f(x)\ dx\,.$

A. 70

B. -10

C. -14

D. 10

E. 2

12. (4 points) If the conclusion of the Mean Value Theorem is applied to the function $f(x) = \dfrac{1}{x-1}$ on the interval $[2, 5]$, which of the following values of c is correct?

A. $c = 1 + \frac{2}{\sqrt{3}}$

B. $c = \frac{7}{3}$

C. $c = 3$

D. c does not exist

E. None of the above.

13. (4 points) Which of these equations is the solution to the initial value problem

$$y' = \sin x, \quad y(0) = 2$$

A. $y = -\cos(x) + 3$

B. $y = -\cos(x) + 2$

C. $y = \cos(x) + 3$

D. $y = \cos(x) + 1$

E. $y = \cos(x)$

14. (4 points) Which of the following is the equation of a horizontal asymptote for the curve $y = \dfrac{3x+\sqrt{x}}{x(1-2x)}$?

A. $y = 0$

B. $y = 3$

C. $y = 3/2$

D. $y = -3/2$

E. $y = -3$

15. (4 points) $\displaystyle\int_0^3 |x-1|\,dx =$

A. 3

B. $\frac{9}{2}$

C. $\frac{5}{2}$

D. 5

E. $\frac{7}{2}$

16. (4 points) Which of the following definite integrals is equivalent to the following limit of a Riemann sum?

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt[3]{2+\frac{4i}{n}} \cdot \frac{4}{n}$$

A. $\displaystyle\int_2^6 \sqrt[3]{2+x}\,dx$

B. $\displaystyle\int_2^6 \sqrt[3]{x}\,dx$

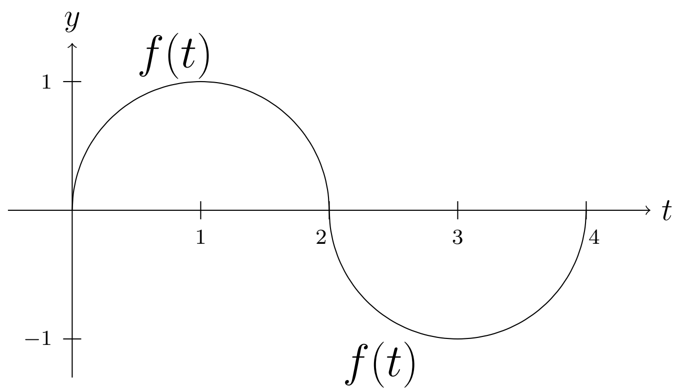
C. $\displaystyle\int_2^6 \sqrt[3]{2+4x}\,dx$

D. $\displaystyle\int_2^6 4\sqrt[3]{2+4x}\,dx$

E. None of the above.

More Challenging Question(s). Show all work to receive credit.

17. Below is the graph of the function $\mathbf{y} = \mathbf{f}(\mathbf{t})$ and is formed by two adjacent semi-circles of radius 1:



Consider the function $g(x)$ defined by $g(x) = \displaystyle\int_0^x f(t)\, dt$, for all $0 \leq x \leq 4$.

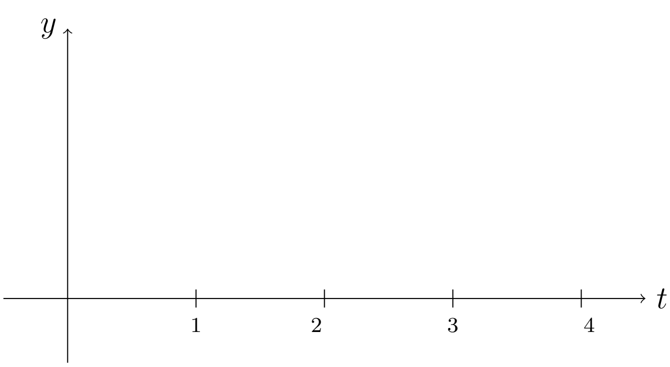
(a) (2 points) Calculate: $\boxed{\text{g(0)=}\quad}$, $\boxed{\text{g(1)=}\quad}$, $\boxed{\text{g(2)=}\quad}$, $\boxed{\text{g(3)=}\quad}$ and $\boxed{\text{g(4)=}\quad}$

(b) (2 points) List the critical point(s) of $g(x)$ and determine if each is a local minimum, a local maximum or neither for g.

(c) (2 points) $g(x)$ achieves its global maximum in the interval $[0, 4]$ at $\boxed{x = \quad}$

(d) (4 points) List the inflection points of $g(x)$.

(e) (4 points) Sketch the graph of $g(x)$.



Congratulations you are now done with the exam!
Go back and check your solutions for accuracy and clarity. Make sure your final answers are $\boxed{\text{BOXED}}$.

When you are completely happy with your work please bring your exam to the front to be handed in.
Please have your MSU student ID ready so that is can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	14	
4	14	
5	14	
6	12	
7	12	
8	12	
9	14	
Total:	106	

No more than 100 points may be earned on the exam.

FORMULA SHEET

Algebraic

- $a^2 - b^2 = (a-b)(a+b)$
- $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
- Quadratic Formula: $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Geometric

- Area of Circle: πr^2
- Circumference of Circle: $2\pi r$
- Circle with center (h,k) and radius r:
$$(x-h)^2 + (y-k)^2 = r^2$$
- Distance from (x_1,y_1) to (x_2,y_2):
$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$
- Area of Triangle: $\frac{1}{2}bh$
- $\sin\theta = \dfrac{\text{opposite leg}}{\text{hypotenuse}}$
- $\cos\theta = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$
- $\tan\theta = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$
- If $\triangle ABC$ is similar to $\triangle DEF$ then
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$
- Volume of Sphere: $\frac{4}{3}\pi r^3$
- Surface Area of Sphere: $4\pi r^2$
- Volume of Cylinder/Prism: (height)(area of base)
- Volume of Cone/Pyramid: $\frac{1}{3}$(height)(area of base)

Trigonometric

- $\sin^2\theta + \cos^2\theta = 1$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
  $= 1 - 2\sin^2\theta$
  $= 2\cos^2\theta - 1$
- Table of Trig Values

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin(x)$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos(x)$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan(x)$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	DNE

Limits

- $\lim_{x\to a} f(x)$ exists if and only if $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$
- $\lim_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$
- $\lim_{\theta\to 0} \dfrac{1-\cos\theta}{\theta} = 0$

Derivatives

- $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$
- $(\cot x)' = -\csc^2 x$
- $(\csc x)' = -\csc x \cdot \cot x$

Theorems

- (IVT) If f is continuous on $[a,b]$, $f(a) \neq f(b)$, and N is between $f(a)$ and $f(b)$ then there exists $c \in (a,b)$ that satisfies $f(c) = N$.
- (MVT) If f is continuous on $[a,b]$ and differentiable on (a,b) then there exists $c \in (a,b)$ that satisfies $f'(c) = \dfrac{f(b)-f(a)}{b-a}$.
- (FToC P1) If $F(x) = \displaystyle\int_a^x f(t)\,dt$ then $F'(x) = f(x)$.

Other Formulas

- Newton's Method: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$
- $\displaystyle\sum_{i=1}^{n} c = cn$
- $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
- $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$