Name: ______________________________

Section: ______________ **Recitation Instructor**: ______________________

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 10.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand.
- You will be given exactly 90 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.

ACADEMIC HONESTY

- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page. You must indicate if you desire work on the back of a page to be graded.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions and statements regarding academic honesty: ______________________________

SIGNATURE

Standard Response Questions. Show all work to receive credit. Please **BOX** your final answer.

1. (4 points) Find the most general antiderivative of $f(x) = \dfrac{2}{x^2} + \sec x \tan x$.

$$\begin{aligned} solution: F(x) &= -2x^{-1} + \sec x + C \\ &= -\frac{2}{x} + \sec x + C \end{aligned}$$

2. (4 points) Evaluate: $\displaystyle\int_0^2 (3x^2 + 5x - 3)\, dx$.

$$\begin{aligned} solution: &= x^3 + \tfrac{5}{2}x^2 - 3x\Big|_0^2 \\ &= \left[(2)^3 + \tfrac{5}{2}(2)^2 - 3(2)\right] - [0] \\ &= 8 + 10 - 6 = 12 \end{aligned}$$

3. (6 points) Solve the initial value problem: $\dfrac{dy}{dx} = -6\sin x, \qquad y(0) = 4$

$$\begin{aligned} solution: y &= 6\cos x + C \\ y(0) = 4 &= 6\cos(0) + C \\ 4 &= 6 + C \\ C &= -2 \\ y &= 6\cos x - 2 \end{aligned}$$

4. (6 points) Find the absolute maximum and absolute minimum values of

$$f(x) = x^3 - 12x$$

on the interval $[-5, 2]$.

$$\begin{aligned} solution : f'(x) &= 3x^2 - 12 \\ 0 &= 3x^2 - 12 \\ x &= \pm\sqrt{\tfrac{12}{3}} = \pm 2 \\ f(-5) &= -65 \\ f(-2) &= 16 \\ f(2) &= -16 \end{aligned}$$

$$\begin{aligned} &\text{Abs. Max is 16 } (@x = -2) \\ &\text{Abs. Min is -65 } (@x = -5) \end{aligned}$$

5. Estimate the area under the graph of $f(x) = -x^2 + 8x + 9$ from $x = 0$ to $x = 8$ using the areas of 4 rectangles of equal width.

(a) (4 points) Estimate the area using left endpoints.

$$\begin{aligned} \Delta x &= 2 \\ LHS &= 2\left(f(0) + f(2) + f(4) + f(6)\right) \\ &= 2(9 + 21 + 25 + 21) = 152 \end{aligned}$$

vfill

(b) (4 points) Estimate the area using an upper sum (overestimate).

$$\begin{aligned} \Delta x &= 2 \\ US &= 2\left(f(2) + f(4) + f(4) + f(6)\right) \\ &= 2(21 + 25 + 25 + 21) = 184 \end{aligned}$$

6. Suppose: $f(x) = \dfrac{3x^2}{x^2-4}$, $f'(x) = \dfrac{-24x}{(x^2-4)^2}$, $f''(x) = \dfrac{24(3x^2+4)}{(x^2-4)^3}$

(a) (4 points) Write the equations for all vertical, horizontal, and slant asymptotes, if they exist.

Horizontal Asymptote: $y = 3$
Vertical Asymptotes: $x = 2, x = -2$

(b) (4 points) Identify the intervals over which $f(x)$ is increasing / decreasing. *Write your answer in interval notation in the boxes below.*

f is increasing on: $(-\infty, -2) \cup (-2, 0]$

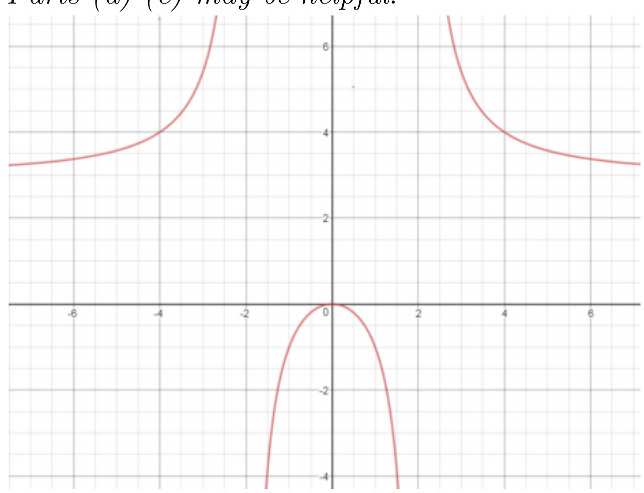
f is decreasing on: $[0, 2) \cup (2, \infty)$

(c) (4 points) Identify the intervals over which $f(x)$ is concave up / concave down. *Write your answer in interval notation in the boxes below.*

f is concave up on: $(-\infty, -2) \cup (2, \infty)$

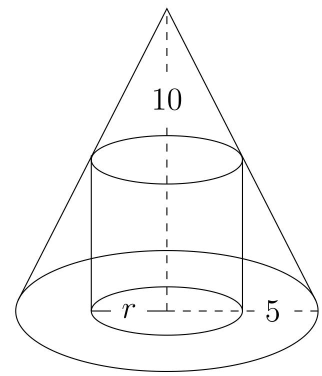
f is concave down on: $(-2, 2)$

(d) (2 points) Sketch the curve of $y = f(x)$. *Parts (a)-(c) may be helpful.*



7. A cylinder is inscribed in a right circular cone with a height of 10cm and a radius (at the base) of 5cm.

(a) (6 points) Express the volume of the cylinder V in terms of its radius r only.



$V = \pi r^2 h$
$h = -2r + 10$

$V(r) = -2\pi r^3 + 10\pi r^2$

(b) (8 points) Find the maximum volume of such a cylinder. *Include units!*

Use techniques of calculus to justify that your answer is a maximum.

$$V'(r) = -6\pi r^2 + 20\pi r$$
$$0 = -6\pi r^2 + 20\pi r$$
$$0 = 2\pi r(-3r + 10)$$
$$r = 0$$
$$r = \frac{10}{3}$$
$$V\left(\frac{10}{3}\right) = -2\pi\left(\frac{10}{3}\right)^3 + 10\pi\left(\frac{10}{3}\right)^2$$

*use number line or other reasoning to justify that r = $\frac{10}{3}$ is a max

8. (4 points) Use a linear approximation to estimate $\sqrt{17}$.

   A. $4+\sqrt{17}$

   B. $4+\frac{1}{2}$

   C. $4+\frac{1}{4}$

   D. $4+\frac{1}{8}$

   E. $4+\frac{1}{16}$

9. (4 points) If $f(x)=x^2-3x$, which of the following statements is true by the Mean Value Theorem?

   A. There is a value c in the interval $(0,4)$ such that $f(c)=1$.

   B. There is a value c in the interval $(0,4)$ such that $f'(c)=1$.

   C. There is a value c in the interval $(0,4)$ such that $f(c)=4$.

   D. There is a value c in the interval $(0,4)$ such that $f'(c)=4$.

   E. The Mean Value Theorem cannot be applied.

10. (4 points) What is the horizontal asymptote of $f(x)=\dfrac{(3x-1)(x-2)}{(x+1)(2x)}$?

   A. The function does not have a horizontal asymptote.

   B. $y=3$

   C. $x=3$

   D. $y=\frac{3}{2}$

   E. $x=\frac{3}{2}$

11. (4 points) Let $F(x) = \int_x^{x^2} \frac{1}{t^2+1}\, dt$. Find $F'(x)$.

A. $F'(x) = \frac{1}{x^4+1}$

B. $F'(x) = \frac{2x}{x^4+1}$

C. $F'(x) = \frac{x^2}{\frac{1}{3}x^6 + x^2} - \frac{3}{\frac{1}{3}x^3 + x}$

D. $F'(x) = \frac{2x}{x^4+1} - \frac{1}{x^2+1}$

E. $F'(x) = \frac{1}{x^4+1} - \frac{1}{x^2+1}$

12. (4 points) Calculate: $\sum_{i=1}^{3} 2^i$.

A. 2

B. 6

C. 8

D. 14

E. 20

13. (4 points) The graph of a function $f(x)$ is shown below. What is the value of $\int_0^3 f(x)\, dx$?

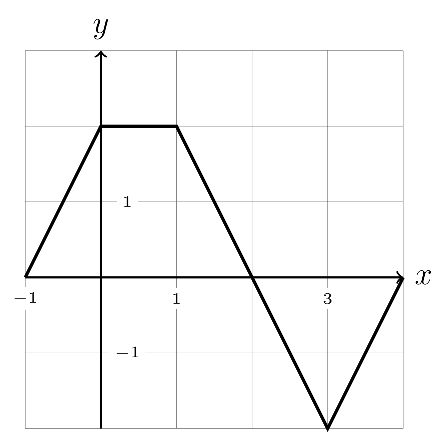
A. 0

B. 1

C. 2

D. 3

E. 4



14. (4 points) Calculate $\int_0^5 |x-2|\, dx$. *Hint: Sketch a graph.*

A. $\frac{13}{2}$

B. $\frac{25}{2} - 10$

C. $\frac{5}{2}$

D. $\frac{3}{2}$

E. None of the above

15. (4 points) Which of the following functions has a critical point where its derivative is undefined?

A. $a(x) = \dfrac{1}{x-1}$

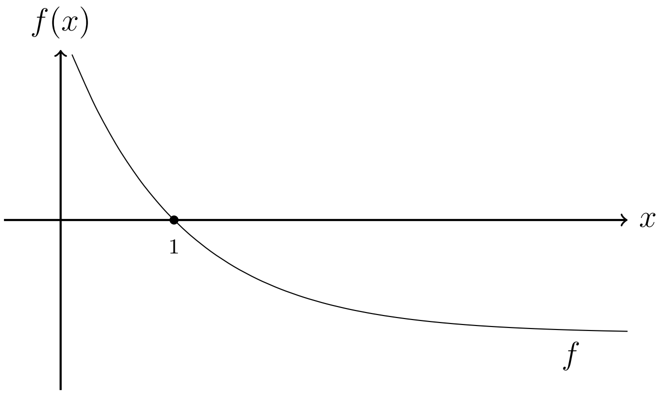
B. $b(x) = \dfrac{x}{(x+1)^2}$

C. $c(x) = x^{2/3}$

D. $d(x) = x^2 + 4x + 1$

E. $e(x) = \sin x$

16. (4 points) The graph of a twice-differentiable function f is shown in the figure below. Which of the following is true?



A. $f(1) < f'(1) < f''(1)$

B. $f(1) < f''(1) < f'(1)$

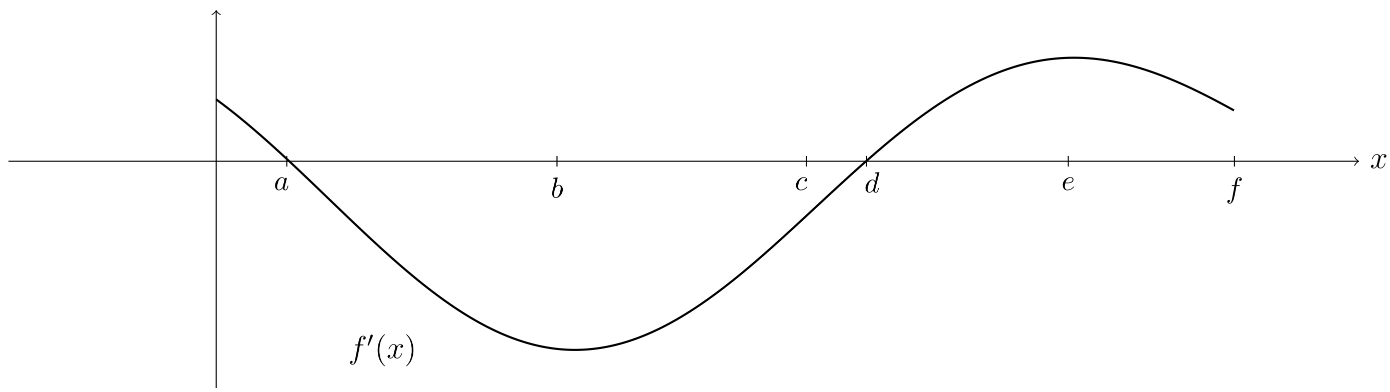
C. $f'(1) < f(1) < f''(1)$

D. $f''(1) < f(1) < f'(1)$

E. $f''(1) < f'(1) < f(1)$

More Challenging Question(s). Show all work to receive credit.

17. The graph of the first derivative $f'(x)$ of a function $f(x)$ is shown below.

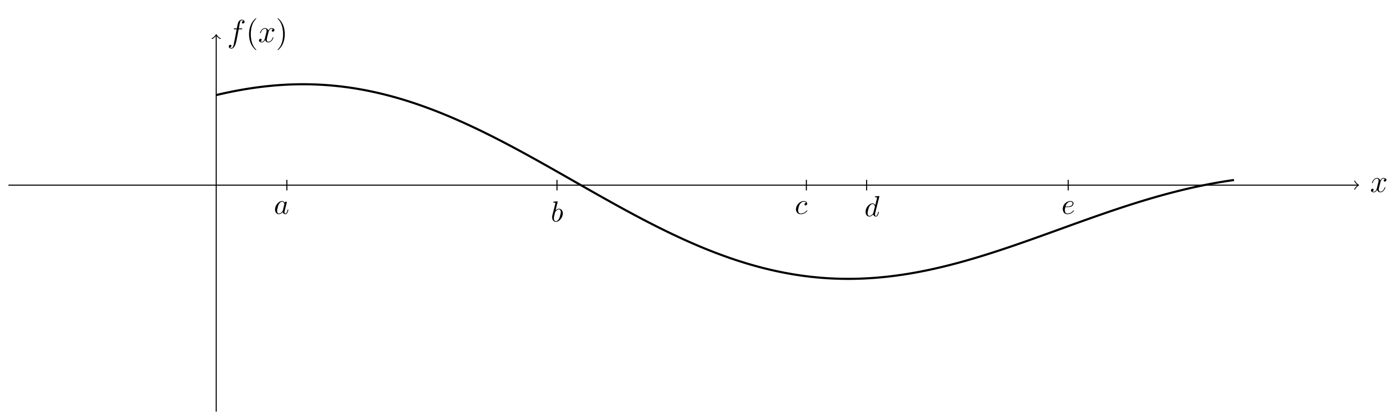


(a) (4 points) List the critical points of $f(x)$ and determine if each is a local maximum, local minimum, or neither.
$x = a$ local max; $x = d$ local min

(b) (4 points) List the inflection points of $f(x)$.
$x = b$; $x = e$

(c) (2 points) *(Circle one)* **True** / **False** $f(b) > f(d)$. **TRUE**

(d) (4 points) Sketch a graph of $f(x)$ given that $f(0) = 1$.



Congratulations you are now done with the exam!
Go back and check your solutions for accuracy and clarity. Make sure your final answers are BOXED.

When you are completely happy with your work please bring your exam to the front to be handed in. **Please have your MSU student ID ready** so that is can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	14	
4	14	
5	14	
6	12	
7	12	
8	12	
9	14	
Total:	106	

No more than 100 points may be earned on the exam.