Name: ____________________

Section: ____________ **Recitation Instructor**: ____________________

INSTRUCTIONS

- Fill in your name, etc. on this first page.
- Without fully opening the exam, check that you have pages 1 through 11.
- **Show all your work** on the standard response questions. Write your answers clearly! Include enough steps for the grader to be able to follow your work. Don't skip limits or equal signs, etc. Include words to clarify your reasoning.
- Do first all of the problems you know how to do immediately. Do not spend too much time on any particular problem. Return to difficult problems later.
- If you have any questions please raise your hand.
- You will be given exactly 90 minutes for this exam.
- Remove and utilize the formula sheet provided to you at the end of this exam.

ACADEMIC HONESTY

- Do not open the exam booklet until you are instructed to do so.
- Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only the proctor(s).
- Books, notes, calculators, phones, or any other electronic devices are not allowed on the exam. Students should store them in their backpacks.
- No scratch paper is permitted. If you need more room use the back of a page. You must indicate if you desire work on the back of a page to be graded.
- Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty can be very severe. All cases of academic dishonesty will be reported immediately to the Dean of Undergraduate Studies and added to the student's academic record.

I have read and understand the above instructions and statements regarding academic honesty: ____________________

SIGNATURE

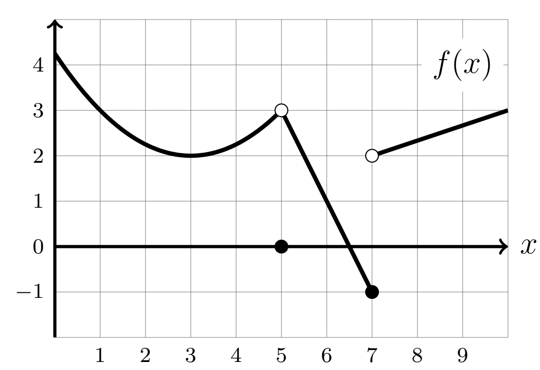
Standard Response Questions. Show all work to receive credit. Please $\boxed{\textbf{BOX}}$ your final answer.

1. (8 points) Calculate the following limits.

(a) $\displaystyle\lim_{x\to 1} \frac{x^2-1}{x^2-4x+3}$

(b) $\displaystyle\lim_{x\to 2} \frac{x+3}{(x-2)^2}$

2. (6 points) Use the graph of $f(x)$ below to calculate the following limits.



(a) $\displaystyle\lim_{x\to 5} f(x) =$

(b) $\displaystyle\lim_{x\to 7^-} f(x) =$

(c) $\displaystyle\lim_{x\to 7} f(x) =$

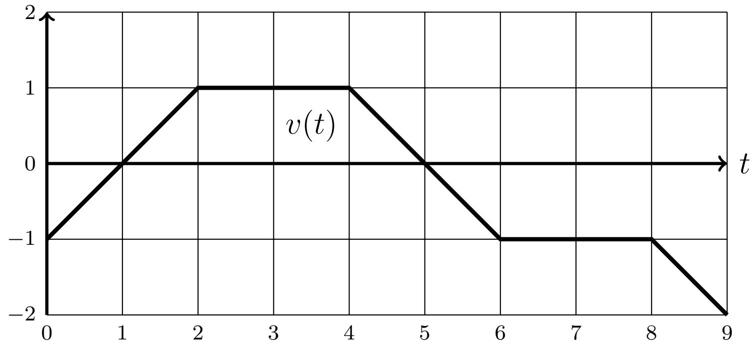
3. (8 points) Calculate the derivative of the following functions.

   (a) $f(x) = 4x^3 + \sqrt[3]{x^2} + \dfrac{2}{x^2} + \sec x$

   (b) $h(x) = \sin\left(x^2 + 1\right)$

4. (6 points) $f(x) = \sqrt{x-3}$. Use the **limit definition** of derivative to show that $f'(x) = \dfrac{1}{2\sqrt{x-3}}$

5. (10 points) The graph below shows the **velocity** *v(t)* of particle moving in a straight line.



(a) What is the maximum speed? (Recall: speed = |velocity|).

(b) What is the maximum velocity?

(c) When is the particle moving forward (i.e. in the positive direction). *Use interval notation.*

(d) When is the particle speeding up? *Use interval notation.*

(e) When is the particle moving at a constant velocity? *Use interval notation.*

6. (4 points) Use the definition of continuity to determine if $f(x)$ is continuous at $x = 1$. State your conclusion and explain your reasoning.

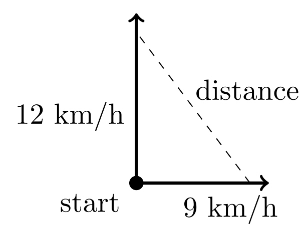
$$f(x) = \begin{cases} x^2 + 2 & x < 1 \\ 4 & x = 1 \\ 2x + 1 & x > 1 \end{cases}$$

7. Suppose that y and x satisfy the implicit equation $y^2 = 3xy - x^3$.

   (a) (6 points) Find the the derivative $\frac{dy}{dx}$.

   (b) (2 points) Find an equation of the tangent line through the point $(-4, 4)$.

8. (6 points) Two boats start sailing from the same point. One boat travels north at 12 km/h and the other travels east at 9 km/h. At what rate is the distance between the boats increasing two hours later?



Multiple Choice. Circle the best answer. No work needed. No partial credit available.

9. (4 points) Suppose $f(x) = |x+3|$. For which value of x is f not differentiable?

    A. $x = 3$

    B. $x = 1$

    C. $x = -3$

    D. $x = 0$

    E. f is differentiable everywhere.

10. (4 points) Suppose $f(x)$ is a continuous function with values given by the table below.

x	0	1	2	3	4	5
$f(x)$	10.1	3.4	2.9	-1.5	0	0.8

In which interval must there be a c for which $f(c) = 3$?

A. $(0, 1)$ B. $(1, 2)$ C. $(2, 3)$ D. $(3, 4)$ E. $(4, 5)$

11. (4 points) If $f(x)$ is a differentiable function, which of the following statements about $f'(1)$ is true?

    (I) $f'(1)$ is the y-value at $x = 1$.

    (II) $f'(1)$ is the average rate of change at $x = 1$.

    (III) $f'(1)$ is the instantaneous rate of change at $x = 1$.

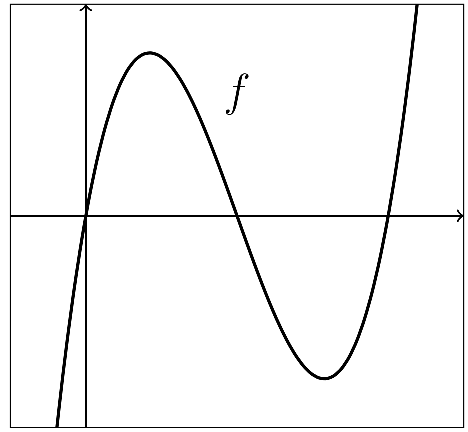
    (IV) $f'(1)$ is the slope of the secant line at $x = 1$.

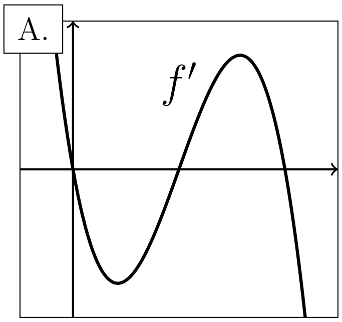
    (V) $f'(1)$ is the slope of the tangent line at $x = 1$.

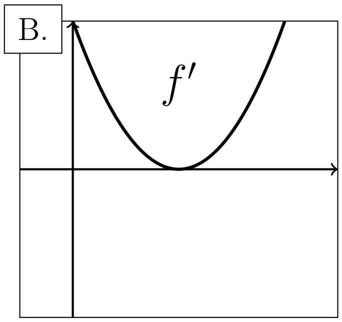
A. (I) only B. (II) only C. (III) only D. (II) and (IV) E. (III) and (V)

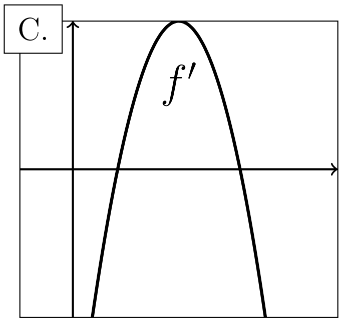
12. (4 points) Consider the graph $y = f(x)$ shown to the right.

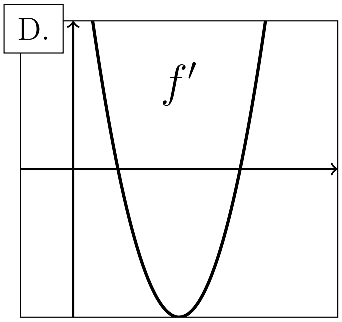
Which of the following is a graph of its derivative?



A.


B.


C.


D.


13. (4 points) Let $f(x) = \begin{cases} x^2, & x \leq 2 \\ 2x, & x > 2 \end{cases}$

Which of the following statements is true?

A. The function f is differentiable, but not continuous, at $x = 2$.

B. The function f is continuous and differentiable at $x = 2$.

C. The function f is undefined at $x = 2$.

D. The function f is neither continuous nor differentiable at $x = 2$.

E. The function f is continuous, but not differentiable, at $x = 2$.

14. (4 points) Calculate: $\lim\limits_{x \to 1^-} \dfrac{x^2 - 1}{|x - 1|}$

A. 2

B. 1

C. $\frac{0}{0}$

D. -2

E. DNE

15. (4 points) Determine the average rate of change of the function $f(t) = 2 + \sin t$ over the interval $\left[0, \frac{\pi}{2}\right]$.

A. $\frac{3\sqrt{3}}{\pi}$

B. $-\frac{1}{\pi}$

C. $\frac{\pi}{12}$

D. $\frac{2}{\pi}$

E. 0

16. (4 points) The velocity $v(t)$ of a particle is shown in the graph below. Which of the following statements is true about the particle motion at $t = 2$?

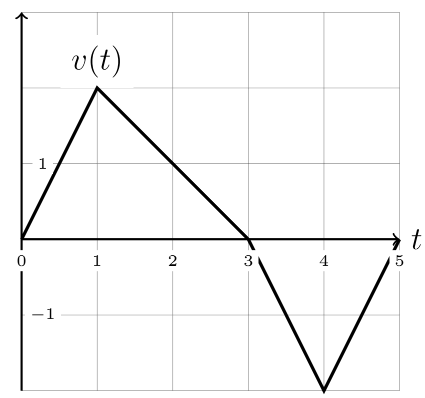
A. The particle is moving backward and speeding up at $t = 2$.

B. The particle is moving backward and slowing down at $t = 2$.

C. The particle is moving forward and speeding up at $t = 2$.

D. The particle is moving forward and slowing down at $t = 2$.

E. None of the above.



17. (4 points) Let $f(1) = 2$, $f'(1) = 3$, $g(1) = 4$, and $g'(1) = 5$. Calculate the following derivative:

$$\frac{d}{dx}\frac{f(x)}{1+g(x)}\bigg|_{x=1}$$

A. $\frac{1}{5}$

B. $-\frac{1}{5}$

C. $-\frac{5}{36}$

D. $\frac{5}{36}$

E. None of the above.

More Challenging Question(s). Show all work to receive credit.

18. (6 points) Calculate: $\lim_{x\to 0^+} \dfrac{\sin\sqrt{x}}{\sin(4x)}$

19. (6 points) Calculate the derivative of $f(x) = \sin^2\left(\dfrac{\tan x}{x^3}\right)$.

20. (2 points) **True** or **False** (*circle one*): $\dfrac{d}{dx}(\pi^4) = 4\pi^3$

Congratulations you are now done with the exam!
Go back and check your solutions for accuracy and clarity. Make sure your final answers are $\boxed{\text{BOXED}}$.

When you are completely happy with your work please bring your exam to the front to be handed in. **Please have your MSU student ID ready** so that is can be checked.

DO NOT WRITE BELOW THIS LINE.

Page	Points	Score
2	14	
3	14	
4	14	
5	14	
6	12	
7	12	
8	12	
9	14	
Total:	106	

No more than 100 points may be earned on the exam.

FORMULA SHEET

Algebraic

- $a^2 - b^2 = (a-b)(a+b)$
- $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
- Quadratic Formula: $\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Geometric

- Area of Circle: πr^2
- Circumference of Circle: $2\pi r$
- Circle with center (h,k) and radius r:
$$(x-h)^2 + (y-k)^2 = r^2$$
- Distance from (x_1, y_1) to (x_2, y_2):
$$\sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$
- Area of Triangle: $\frac{1}{2}bh$
- $\sin\theta = \dfrac{\text{opposite leg}}{\text{hypotenuse}}$
- $\cos\theta = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$
- $\tan\theta = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$
- If $\triangle ABC$ is similar to $\triangle DEF$ then
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$
- Volume of Sphere: $\frac{4}{3}\pi r^3$
- Surface Area of Sphere: $4\pi r^2$
- Volume of Cylinder/Prism: (height)(area of base)
- Volume of Cone/Pyramid: $\frac{1}{3}$(height)(area of base)

Theorems

- (IVT) If f is continuous on $[a,b]$, $f(a) \neq f(b)$, and N is between $f(a)$ and $f(b)$ then there exists $c \in (a,b)$ that satisfies $f(c) = N$.

Limits

- $\lim\limits_{x\to a} f(x)$ exists if and only if $\lim\limits_{x\to a^-} f(x) = \lim\limits_{x\to a^+} f(x)$
- $\lim\limits_{\theta\to 0} \dfrac{\sin\theta}{\theta} = 1$
- $\lim\limits_{\theta\to 0} \dfrac{1-\cos\theta}{\theta} = 0$

Derivatives

- $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$
- $(\cot x)' = -\csc^2 x$
- $(\csc x)' = -\csc x \cdot \cot x$

Trigonometric

- $\sin^2\theta + \cos^2\theta = 1$
- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta$
$= 1 - 2\sin^2\theta$
$= 2\cos^2\theta - 1$
- Table of Trig Values

x	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\sin(x)$	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1
$\cos(x)$	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0
$\tan(x)$	0	$\sqrt{3}/3$	1	$\sqrt{3}$	DNE